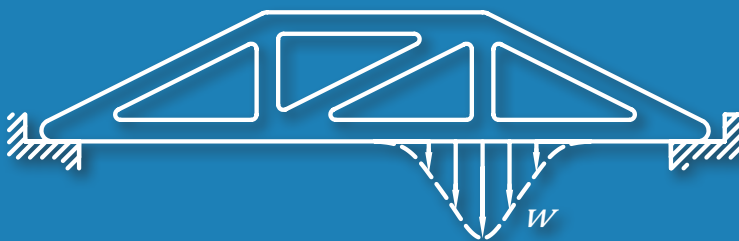
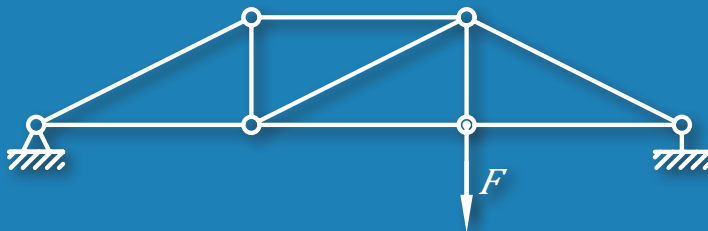


Oleksii Nosko

STRENGTH OF STRUCTURES

Lecture notes and study problems



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Contents

Preface	5
Notation	6
Schematic designations	9
Lecture 1. Introduction and terminology	13
Lecture 2. Geometry of cross sections	25
Problem group P1. Static moments and centroid	36
Problem group P2. Moments of inertia	42
Problem group P3. Principal moments of inertia	48
Lecture 3. Tension and compression	54
Problem group P4. Stresses in an axially loaded bar	66
Problem group P5. Displacements in an axially loaded bar	72
Problem group P6. Stresses in a pin-joint truss	79
Problem group P7. Bar structures with a rigid element	87
Lecture 4. Torsion	93
Problem group P8. Torsion of a circular shaft	100
Lecture 5. Bending stresses	106
Problem group P9. Bending moments in a beam	118
Problem group P10. Stresses in a beam	132
Lecture 6. Bending deflections	144
Problem group P11. Deflections and slopes in a beam	152
Lecture 7. Combined loading	161
Problem group P12. Combined loading on a cross section	169
Problem group P13. Bending stresses in a plane frame	177
Lecture 8. Strength theories	183
Problem group P14. Equivalent stress	193
Problem group P15. Stresses in a frame	199
Lecture 9. Energy theorems	208
Problem group P16. Strain energy	215
Problem group P17. Displacements in a structure	221

Lecture 10. Force method	228
Problem group P18. Statically indeterminate beams	237
Problem group P19. Statically indeterminate plane frames	243
Problem group P20. Statically indeterminate frames	249
Remarks on the use of artificial intelligence	256
Recommended literature	257
Abstract	258
Index	259

Preface

This book is a collection of lecture notes and study problems on the subject of *strength of materials*, also known as *mechanics of materials* or *mechanics of deformable bodies*. It is designed to teach students the most important terms, assumptions, principles and methods of the subject.

The content of the book is organised in the form of ten lectures. Lectures **1–6** are dedicated to the problems of tension, compression, torsion and bending. Lectures **7** and **8** deal with the problems of plane stress under combined loading. Lectures **9** and **10** introduce energy methods for solving statically determinate and indeterminate problems.

Before taking a course in *strength of materials*, students should have completed a course in *mechanics*. In particular, they are expected to know and be able to apply the conditions of static equilibrium and the principles of conservation of energy. Since the solution of study problems requires the application of methods of differential and integral calculus, students should also have completed a course in *mathematical analysis*.

The aim of the book is to help students develop skills to efficiently schematise, solve and analyse typical problems of the subject using various approaches. The study problems considered are categorised into twenty groups **P1–P20**, each assigned to a specific lecture. The study problems are formulated both in the ‘classical’ form, where the goal is to determine an internal force, displacement, stress or strain in a structure under the action of certain loads, and in the ‘engineering’ form, where the goal is to determine the maximum loads or minimum geometric sizes for which the structure retains its load-carrying capacity. The problem solutions are found using the analytical methods described in Lectures **1–10**. In addition, the book provides links to the problem definitions as prompts to be solved by artificial intelligence, as well as links to the numerical solutions obtained using finite element analysis. The book contains all the algorithms and formulae necessary to solve the study problems. Nonetheless, it should be considered as a basic introduction to the subject. Systematic reading of comprehensive textbooks, such as [1–6], is necessary for a deeper understanding of the subject.

Finally, I would like to express my gratitude to Prof. Heorhiy Sulym at Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the National Academy of Sciences of Ukraine, Prof. Pavlo Nosko at State University ‘Kyiv Aviation Institute’ and Dr Yuriy Tsybrii at Gdańsk University of Technology for their kind assistance in the preparation of this book. My sincere thanks also go to my family for their support and patience.

Oleksii Nosko

Notation

a	– coefficient; size, diameter, m
b	– coefficient; width, m
d	– diameter, diagonal, distance, m
d_{\max}	– maximum distance from the neutral axis, m
h	– height, m
i, j, k	– indices
k	– spring constant, N/m
m	– intensity of a distributed twisting moment, N
n	– safety factor; degree of static indeterminacy
\bar{n}	– unit normal vector; principal direction
q	– intensity of a distributed axial force, N/m
r	– radius, m
s	– section modulus, m ³
t	– thickness, m
\bar{t}	– stress vector, Pa
v	– beam deflection, m
w	– intensity of a distributed transverse force, N/m
x, y, z	– axes; coordinates, m
A	– cross-sectional area, m ²
C	– centroid; integration constant
C_R	– resultant force couple, Nm
E	– elastic modulus, Pa
F	– force, N
G	– shear modulus, Pa
I	– moment of inertia, m ⁴
I_1, I_2	– principal moments of inertia, m ⁴
I_{xy}	– product of inertia, m ⁴
J	– polar moment of inertia, m ⁴
L	– length, m
M	– moment, bending moment, Nm
M_i	– internal bending moment, Nm
M_{\max}	– maximum bending moment, Nm
N	– normal force, N
P	– axial force, normal force, N

P_{cr}	– critical load, N
P_i	– internal axial force, N
Q	– static moment, m ³ ; dummy load, N or Nm
R	– resultant force, N
T	– twisting moment (torque), Nm
T_i	– internal twisting moment, Nm
U	– strain energy, J
V	– shear force, N
V_{max}	– maximum shear force, N
W	– work, J
X	– redundant reaction, N or Nm
X, Y, Z	– centroidal axes, principal axes, principal directions; coordinates, m
α	– linear thermal expansion coefficient, 1/°C
γ	– shear strain, rad
δ	– elongation, displacement, gap, m
ϵ	– normal strain
ϵ_T	– thermal strain
θ	– angle of twist, rad
λ	– slenderness ratio
ν	– Poisson's ratio
π	– Pi number, $\pi \approx 3.14$
ρ	– radial coordinate, curvature radius, m
σ	– normal stress, Pa
σ'	– complementary normal stress, Pa
$\sigma_1, \sigma_2, \sigma_3$	– principal stresses, Pa
σ_e	– equivalent stress, Pa
σ_{max}	– maximum normal stress, Pa
σ_p	– proportional limit, Pa
σ_{ult}	– ultimate stress, Pa
σ_w	– working stress, Pa
σ_{yp}	– yield stress (yield point), Pa
τ	– shear stress, Pa
τ'	– complementary shear stress, Pa
τ_{max}	– maximum shear stress, Pa
τ_w	– working shear stress, Pa
φ	– angle, angular coordinate, slope angle, angle of rotation, rad
ΔA	– area element, m ²
ΔL	– length element, m
ΔT	– temperature increase, °C
$\text{atan}(\blacksquare)$	– inverse tangent function
$\text{cos}(\blacksquare)$	– cosine function
$\text{max}(\blacksquare)$	– maximum value function
$\text{min}(\blacksquare)$	– minimum value function

$\sin(\blacksquare)$	– sine function
$\tan(\blacksquare)$	– tangent function
$ \blacksquare $	– absolute value function
$H(\blacksquare)$	– unit step function
$\langle \blacksquare \rangle$	– bracket function
\blacksquare	– vector
\blacksquare^t	– transposition of a vector
\blacksquare_i	– i^{th} component of a vector; i^{th} element of a system
$\blacksquare_{\text{max}}$	– maximum value
\blacksquare_x	– related to the axis x
\blacksquare_y	– related to the axis y
\blacksquare_z	– related to the axis z
\blacksquare_c	– related to the centroid
NA	– neutral axis
J	– joule
μm	– micrometre = 10^{-6} m
mm	– millimetre = 10^{-3} m
m	– metre
N	– newton
kN	– kilonewton = 10^3 N
Nm	– newton \times metre
Pa	– pascal = N/m^2
MPa	– megapascal = 10^6 Pa
GPa	– gigapascal = 10^9 Pa
$^{\circ}\text{C}$	– degree Celsius